3D Full-wave Field Simulations of Double Negative Metamaterial Macrostructures

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Abstract

A full-wave 3D numerical analysis methodology for metamaterial structures is presented. Effective description for a double negative unit cell is found by parameter fitting of dispersive models. The obtained models are implemented in the higher level simulations to characterize large metamaterial macrostructures. The simulation results for detailed and effective representation of metamaterial macrostructures are compared and potential savings in numerical costs are pointed out.

1. Introduction

During a few recent years, artificial media with negative real parts of the electric permittivity and magnetic permeability (hence DNG: double negative materials) created enormous interest in the scientific community. Metamaterial (MTM) structures are built of periodically ordered cells, with the assumption that the lattice constant is much shorter than the wavelength in the medium. A DNG cell typically contains fine elements (e.g. narrow gaps in split ring resonators, thin metal wires) in its structure. Consequently, the details of the structure are very small with respect to the wavelength, which makes it challenging to include them in the full-wave 3D electromagnetic analysis. On the other hand, the observation of the desirable macro-effects requires that the DNG structure should have the size of at least several wavelengths (e.g. negative refraction experiment from [1]). When both micro- and macro- requirements are combined, one ends up with the computationally large problem which renders the numerical analysis impractical. To make the simulation of macro-effects possible, one describes a DNG structure by the effective parameters, namely effective electric permittivity and effective magnetic permeability. Effective parameters are typically obtained from the simulation of metamaterial unit cells. The simulation results of the effective macrostructure should be equivalent to the corresponding results for the detailed model.

2. Extraction of effective material parameters

The DNG unit cell used in this work is a combination of a split ring resonator (SRR) and a wire (Fig.1a). The unit cell is simulated with the time-domain solver of CST Microwave

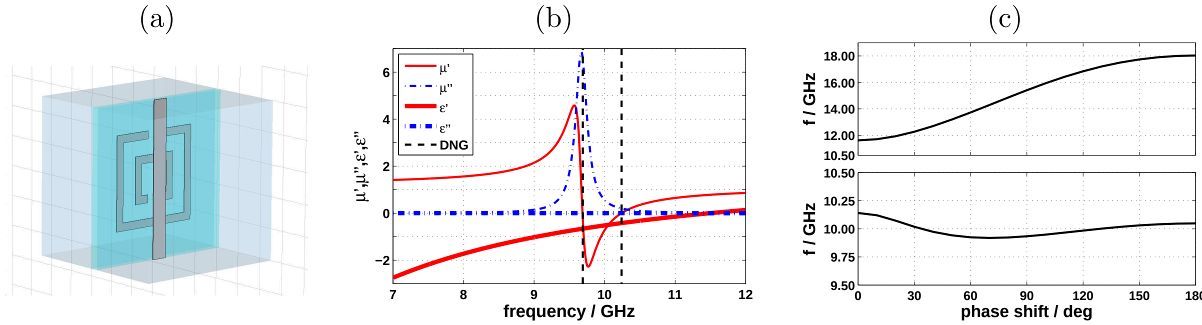

Figure 1: (a) DNG unit cell; (b) effective material parameters obtained by PFDM for the DNG unit cell: $\mu'(\epsilon')$ thin (thick) red solid line, $\mu''(\epsilon'')$ thin (thick) blue dash-dot line; vertical black dashed lines limit the DNG frequency band; (c) dispersion diagram for the DNG unit cell

Studio (MWS) [2], providing frequency dependent scattering parameters treated as the reference response.

The extraction approach is based on the parameter fitting of dispersive models (PFDM) [3]. The main difference from the traditional extraction methods relies on the fact, that the shape of the parameterized characteristics for effective permittivity and permeability is assumed a priori, and their parameters are optimized in order to obtain the best fitting of the corresponding scattering parameters to the reference responses. The obtained electric permittivity and magnetic permeability of the effective unit cell representation are assumed to be of the Drude and Lorentz type, respectively, and are presented in Fig.1b. The structure shows a DNG behavior in the frequency range 9.69-10.24 GHz. The effective description is assumed to be isotropic, which is a simplification valid only for a proper orientation of the fields in the structure (i.e. vertical E and horizontal H components).

In order to confirm the obtained results, the DNG unit cell is simulated independently with MWS eigenmode solver. By applying periodic boundary conditions in the direction of propagation and sweeping the corresponding phase shift, one obtains the dispersion diagram $\omega(\beta)$ given in the Fig.1c. In the frequency range up to 20 GHz, there are 2 modes in the periodic structure composed of the DNG unit cells. There is a fundamental mode propagated in the frequency range 9.91-10.14 GHz and the 1st higher order mode between 11.63 and 18.04 GHz. The fundamental mode represents the backward wave (negative slope of the dispersion curve) for the phase shift up to 90 deg along the unit cell. This corresponds to the maximal unit cell size smaller than $\lambda/4$ and is fulfilled for this MTM structure. The frequency range of the backward wave obtained from eigensolver simulations fits in the DNG frequency range obtained by time-domain simulations and PFDM, which confirms the correctness of the presented approach.

3. Modeling of MTM macrostructures

The modelled macrostructure is a numerical representation of the negative refraction experiment presented by Shelby et al. in [1]. The numerical model of the detailed wedge structure is built of 17x9 MTM unit cells, which results in 89 MTM unit cells. The refraction interface has a staircase pattern with a step ratio of 2:1 which can be referred as a wedge of 26.6 deg. The simulated detailed configuration and the corresponding effective structure are presented in Fig.2a,b. The dimensions of the homogeneous wedge are equivalent to the detailed wedge implementation, while its effective ϵ and μ parameters are described by the Drude and Lorentz dispersive models obtained by PFDM. In Fig.2c the $|S11|$ parameter for both the detailed and effective structure is presented. From the dip in the $|S11|$ characteristic one can see that the transmission for the

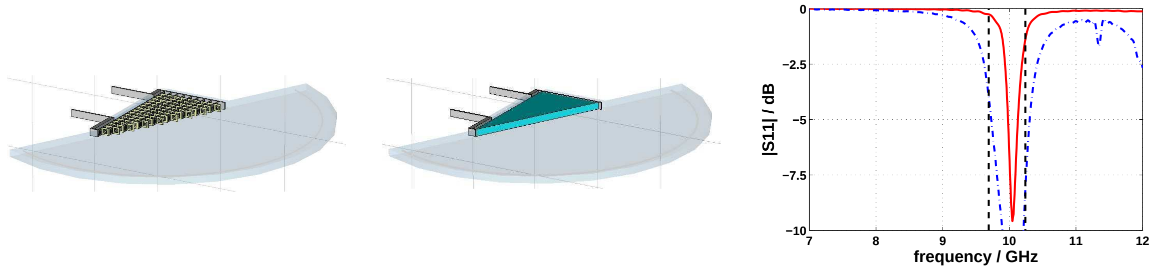

Figure 2: (a/b) detailed/effective wedge structure; (c) $|S11|$ for detailed (red solid line) and effective (blue dash-dot line) DNG wedge simulated in CST Microwave Studio; vertical black dashed lines limit the DNG frequency band

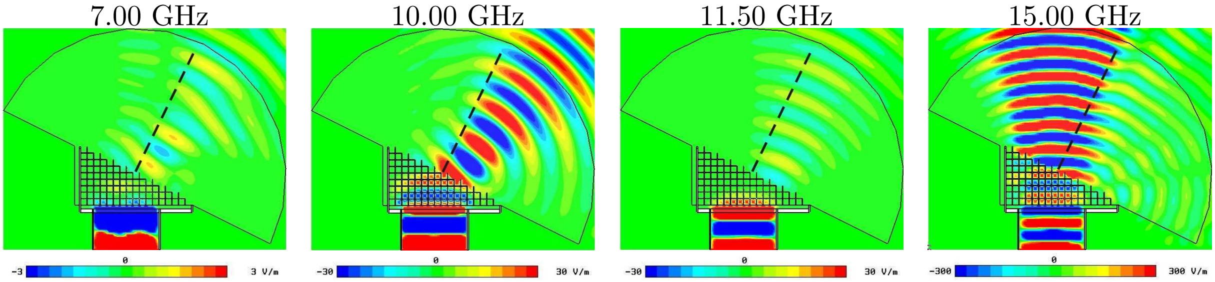

Figure 3: The distribution of the electric field at four frequencies for the detailed wedge structure (dashed line: normal to the boundary between MTM and parallel plate waveguide)

wedge structure takes place in the DNG frequency range obtained from the analysis of the single unit cell.

The distribution of the electric field simulated for the detailed wedge structure is pictured in Fig.3. In the simulated frequency range (2-20 GHz) one can distinguish four frequency regions (cf Fig.1b,c) [4]. The first frequency region up to 9.69 GHz is the stopband in which ϵ is negative while μ is positive (a very weak transmission is observed in this band). The second frequency region from 9.69-10.24 GHz is the double negative band, where negative refraction is observed. The third frequency region 10.24-11.63 GHz is the stop band (negative ϵ and positive μ), characterized by the ultralow refractive index. Weak transmission nearly parallel to the normal of the refractive interface occurs in this band. The fourth frequency region 11.63-18.04 GHz is the positive-index band (both ϵ and μ are positive), where the effective index increases from ultralow to near unity.

The equivalent distribution of the electric field simulated for the effective wedge is presented in Fig.4 (the effective description is valid in the frequency range 7-12 GHz). From the corresponding pictures a good qualitative agreement between the field distributions can be seen.

In Fig.5 the refraction angle θ_R and refractive index n for both the detailed and effective wedge structures are compared. The direction of propagation is found as a maximum in electric field distribution along the semicircular curve of 12cm radius. As the reference the theoretical values based on Snell's law and extracted ϵ and μ (Drude, Lorentz) parameters are given. The corresponding characteristics agree in the DNG frequency range where the negative refractive index can be noticed. Some disagreement between the results for the detailed and effective wedge occurs in the lower part of the DNG range, due to the high losses in this area (cf high values of μ'' in Fig.1b).

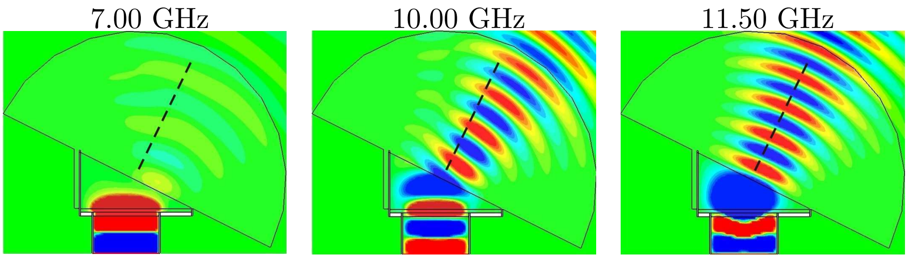

Figure 4: The distribution of the electric field at three frequencies for the effective wedge structure (dashed line: normal to the boundary between MTM and parallel plate waveguide)

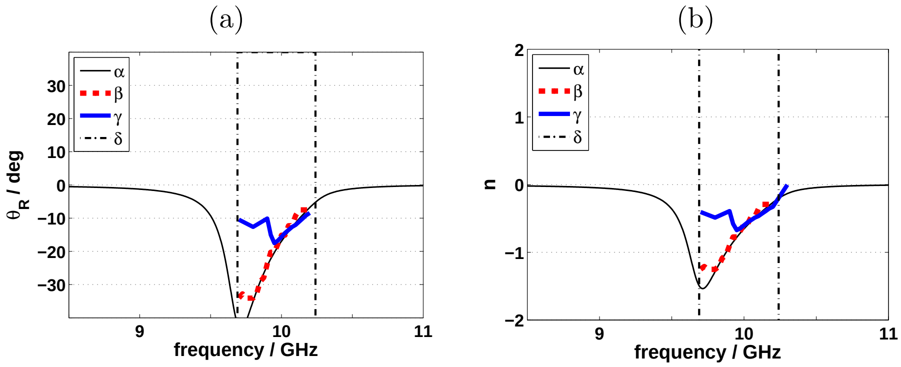

Figure 5: (a) refraction angle, (b) refractive index: from extracted ϵ (Drude model) and μ (Lorentz model) values (α black solid thin line), effective wedge simulation (β red dashed line), detailed wedge simulation (γ blue solid thick line); vertical lines limit the DNG frequency band obtained by PFDM (δ black dash-dot line)

4. Conclusion

A numerical approach to the modelling of metamaterial structures was proposed. Starting with the numerical analysis of a single unit cell, effective material parameters are obtained by parameter fitting of dispersive models. The resulting dispersive models form the effective description of the macrostructure and allow for substantial reduction of numerical costs connected with 3D full-wave numerical analysis of metamaterials (in the analysed example memory requirements are reduced by a factor of 120 and CPU costs by a factor of 80).

References

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