Convergence of the Finite Integration Technique on Various Mesh Types

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***Abstract* — The paper presents a comparison between three widely-used mesh types for 3D electromagnetic simulation, in terms of accuracy, computing speed and memory requirements. Although the numerical technique used here was the Finite Integration Technique, the conclusions can be extrapolated to other numerical methods such as FDTD or FEM.**

I. The Finite Integration Technique

In order to cope with the geometrical and functional diversity of high frequency structures, a variety of methods are used, such as Finite Difference Time Domain – FDTD, finite elements –FEM, integral methods such as the method of moments, etc. Last but not least, the Finite Integration Technique – FIT, first proposed by Weiland in 1977 [1] can be considered as a generalization of the FDTD method and also has tight links to the FEM. It discretizes the integral form of Maxwell's equations (1), rather than the differential one, on a pair of dual interlaced discretization grids.

$$\oint_{\partial A} \vec{E}\cdot d\vec{s} = -\frac{\partial}{\partial t}\iint_A \vec{B}\cdot d\vec{A};\quad \oint_{\partial A} \vec{H}\cdot d\vec{s} = \iint_A \left(\frac{\partial \vec{D}}{\partial t} + \vec{J}\right)\cdot d\vec{A} \qquad (1)$$

The degrees of freedom are also of integral type: electrical voltages and magnetic fluxes, defined on the edges and facets of the primary grid, magnetic voltages and electric fluxes, defined on the edges and facets of the secondary grid (Figure 1).

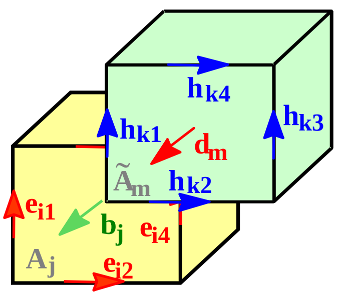


Fig. 1. Dual discretization grids: the grids are interlaced by one-half spatial step. On the primary grid, electric voltages and magnetic fluxes are located.

By writing the integrals on the left sides of (1) as sums of voltages, and by defining the discrete curl-operators (topological matrices) and for the primary and the dual grid respectively, Faraday's and Ampère's Grid Equations become:

$$\mathbf{C}\hat{\mathbf{e}} = -\frac{d}{dt}\hat{\hat{\mathbf{b}}},\qquad \tilde{\mathbf{C}}\hat{\mathbf{h}} = \frac{d}{dt}\hat{\hat{\mathbf{d}}} + \hat{\hat{\mathbf{j}}} \qquad (2)$$

The material property relations become, after discretization:

$$\hat{\hat{\mathbf{d}}} = \mathbf{M}_{\varepsilon}\hat{\mathbf{e}};\quad \hat{\hat{\mathbf{b}}} = \mathbf{M}_{\mu}\hat{\mathbf{h}};\quad \hat{\hat{\mathbf{j}}} = \mathbf{M}_{\sigma}\hat{\mathbf{e}} + \hat{\hat{\mathbf{j}}}_S \qquad (3)$$

Relations (2) have purely topological character and are exact –on a given mesh, while the metric properties and the approximations are contained in the relations (3). This separation has important theoretical, numerical and algorithmic consequences [2].

The FIT can be applied not only to different frequency ranges, from DC to THz, but also on different mesh types (Fig. 2). On Cartesian grids, the time-domain FIT can be shown to be equivalent to FDTD. However, whereas the classical FDTD has the disadvantage of the staircase approximation of complex boundaries, the Perfect Boundary Approximation (PBA)® [3] technique applied in conjunction with FIT maintains all the advantages of the structured Cartesian grids, while allowing an accurate modeling of curved boundaries. Last but not least, the FIT can also be applied on tetrahedral meshes, and it is well known now that the FEM itself ("the" tetrahedron-method) is characterized by equations of the same type, (2) and (3).

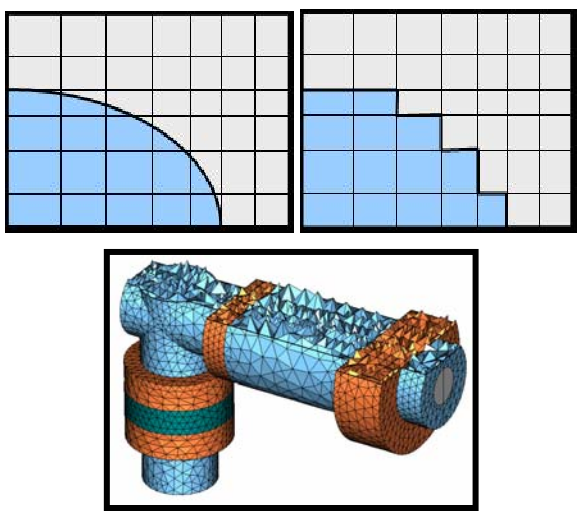


Fig. 2. Examples of PBA, staircase and tetrahedral meshes.

Using the advantage of FIT's applicability on 3 types of meshes, in the present paper we will try to estimate, based on numerical experiments, how much accuracy, memory efficiency and speed can be expected from a simulation based on each of the 3 meshtypes.

II. Numerical Investigation of Convergence Properties

To study convergence, the procedure is well-known and simple: discretize the structure with an increasing number of mesh cells, perform the simulation for each mesh, plot the result (e.g. the error, or the computing time) versus the number of mesh cells, in double-logarithmic scales.

All the simulations presented below were performed with the commercial software package CST MICROWAVE STUDIO® [4].

A. Problem with Analytical Solution

The first test case is a very simple one, with known analytic solution: calculation of the line impedance of a coaxial cable.

A short piece of coax with analytical impedance of 50 Ohm has been modelled and discretized with an increasing number of mesh cells. The error of the line impedance vs. number of mesh cells is shown in Fig. 3.

The plot shows that, in order to reach an impedance accuracy of 1%, the number of necessary mesh cells is of about 700 for the PBA mesh, 250 for the tetrahedral mesh, and 38,000 for the staircase mesh, i.e. 50 times more than PBA !

The final meshes are shown in Fig. 4. A strong refinement in the rounded part of the dielectric, especially at the interior contour, can be noticed in the staircase mesh.

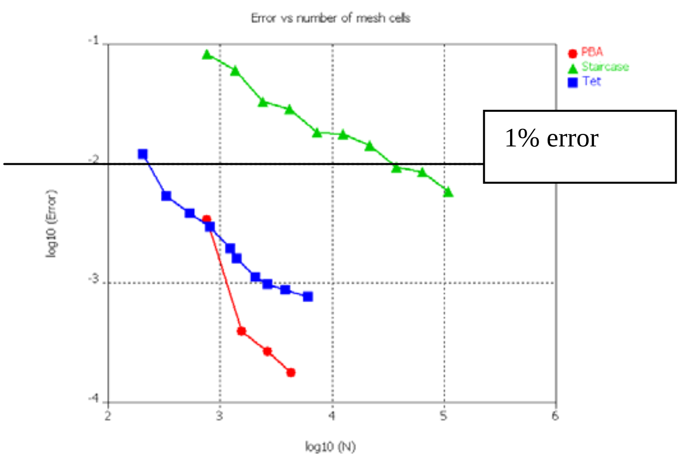


Fig. 3. Error of the line impedance of a coaxial cable vs the number of mesh cells.

B. Coaxial Connector (Moderate-Size Problem)

The second test case was a coaxial connector, shown in Fig. 5. Although the structure looks quite simple, it is a good test vehicle, because it has many rounded parts, both dielectric and metallic, and it exhibits multiple reflections.

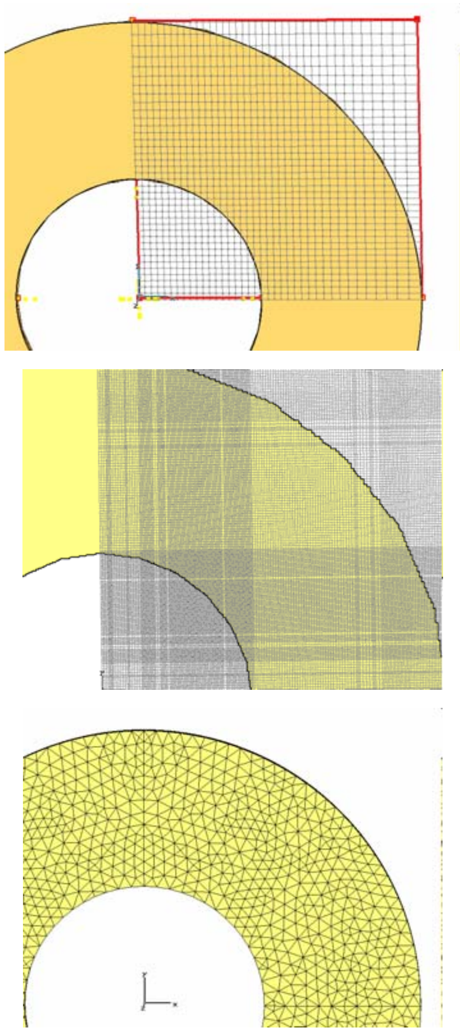


Fig. 4. Coaxial cable impedance simulation: Final mesh at the end of the convergence study, for PBA (4300 meshcells), staircase (110,000 meshcells) and tetrahedral (6175 meshcells) meshes.

The simulation consisted in an adaptive meshing, having as stop criterion a difference of less than 0.01 of the S-parameters between two successive runs.

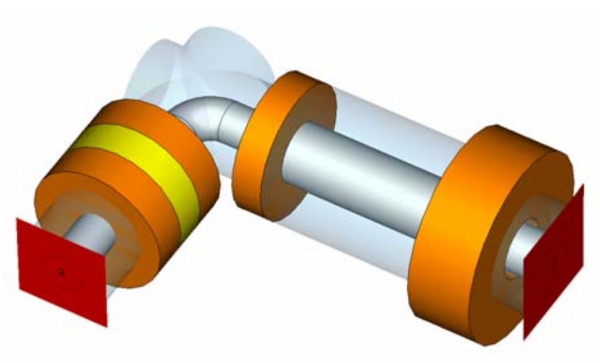


Fig. 5. Coaxial connector test structure.

Fig. 6 shows the variation of the S-parameters' correction and of the computing time with the number of mesh cells. Some interesting things can be noticed:

- The "convergence" for the staircase mesh is very irregular; a large number of mesh cells is necessary for an acceptable solution; as will be shown in the final paper, the staircase mesh did NOT even reach an acceptable solution within this number of passes.

- Although, as expected, the correction varies in an identical manner for both the direct and the iterative solvers on the tetrahedral mesh, the computing time is *substantially* larger when a direct solver is used.

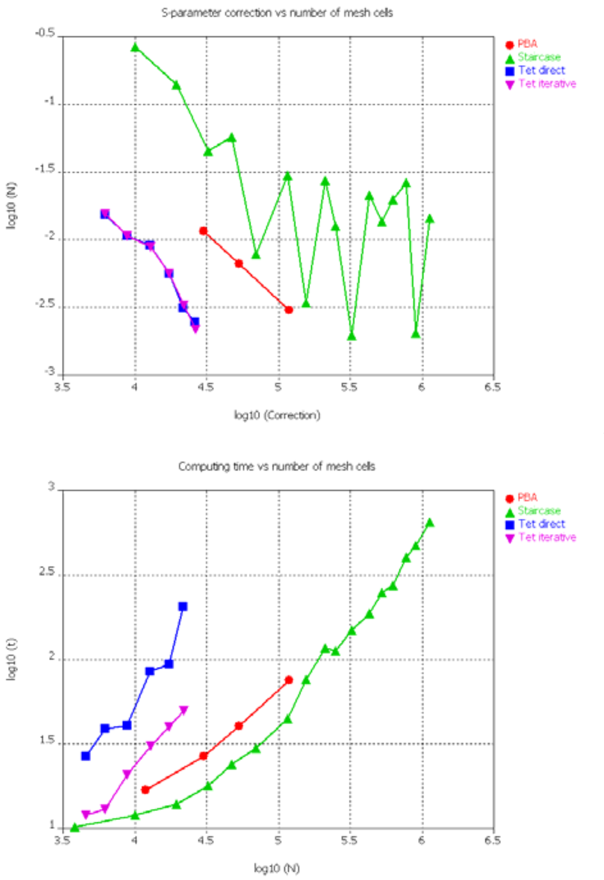


Fig. 6. Coaxial connector results. Upper plot: S-parameter correction vs number of mesh cells; Lower plot: computing time vs number of mesh cells

III. Real Life Applications

Real-life applications are often large or require a large number of simulations (e.g. in an optimization process). In this section, two such application examples are presented. Simulations on various mesh types were performed, and the results, as well as the computing time and the necessary memory are compared.

A. 16-Port Divider (Large Problem)

This structure is a relatively large one, a 16-port power divider whose spatial dimensions are about 16 λ x 14 λ x 0.4 λ (note that this is not yet a *very* large structure!). The geometry of the problem is shown in Fig. 7.

The simulation was performed on the standard mesh (e.g. 10 meshcells / wavelength for PBA and staircase meshes). The purpose of the simulations was to assess the computing time and the necessary memory for large structures. The electric field at 24.76 GHz is shown in Fig. 7, whereas the simulated S-parameters (for excitation at port 1) are shown in Fig. 8.

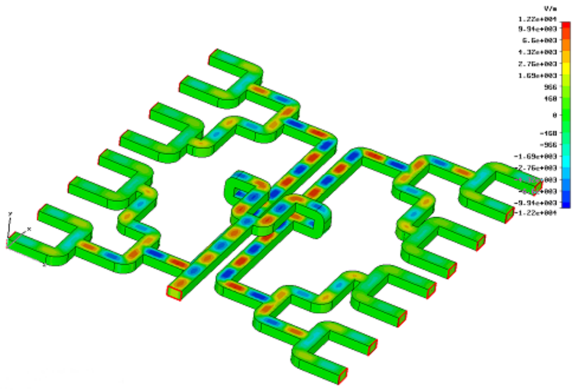


Fig. 7. 16-port power divider

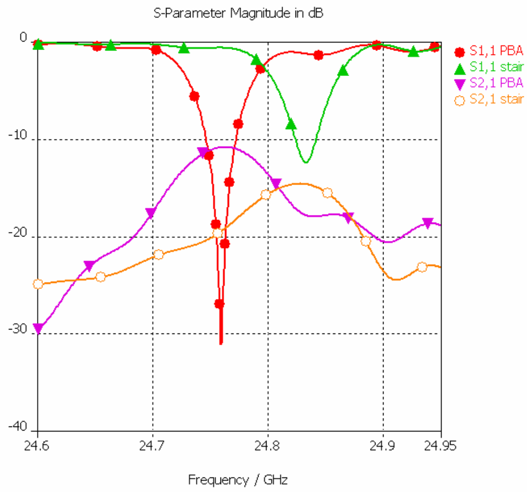


Fig. 8. Simulated S-parameters of the 16-port power divider, for PBA and staircase meshes

The computing time and the necessary memory are summarized in Table 1. Again, the PBA and staircase meshes require the least memory and provide the solution in the shortest time. However, whereas the PBA and the tetrahedral mesh solutions were in good correlation, the results on the staircase mesh did not prove to be satisfactory: a finer mesh is necessary in order to correctly represent the curved details of the structure.

Mesh type	Mesh cells	Comp. time	Memory
PBA	634000	40’	110 MB
Staircase	634000	41’	104 MB
Tet–direct solver	65700	97’	350 MB
Tet – iter. solver	65700	80’	1.1 GB

TABLE I

Simulation of a relatively large structure: computing time and necessary memory

B. Pass-Band Filter

The structure, shown in Fig. 9 is a pass-band, 4-post filter. The purpose of the simulations was to numerically tune (optimize) the filter, finding the dimensions which ensure the prescribed filter characteristics. The tuning procedure is the one described in [5] and is also often used in practice for the final tuning of the filters. Two mesh types were compared, namely PBA and staircase.

filter's dimensions, shown in the lower plot of Fig. 10, are very different from the required ones.

The explanation of this fact is that, due to the poor approximation of the curved geometry, the coupling between the cylindrical cavities / posts cannot be correctly represented in the mesh. With a staircase mesh a filter can thus be "untunable" numerically, despite the excellent properties of the real structure.

IV. Conclusion

A few practical conclusions can be drawn based on the results presented in this paper.

The **staircase mesh** can be very good on structures without curved or slanted boundaries/interfaces and without thin metallic details. Otherwise, it can be said that the solution practically does not converge, or it doesn't converge in a reasonable amount of computing time.

The **PBA mesh** has excellent convergence properties, and allows obtaining a solution in a very short time. The algorithm used in the simulations is an explicit, first-order one, having very low memory requirements, thus allowing the simulation of very large structures.

The **tetrahedral mesh** also exhibits excellent convergence properties. However, even for moderate structures, both the computing time and the memory needed by the direct solver may become prohibitive. For larger structures, an iterative solver is a must.

For very large structures, of tens of wavelengths dimension, the only viable solution can be obtained with the **PBA mesh**.

Note that the conclusions drawn for the staircase mesh are fully valid also for the **FDTD method**, while the ones for the tetrahedral meshes also apply to the **FE method**.

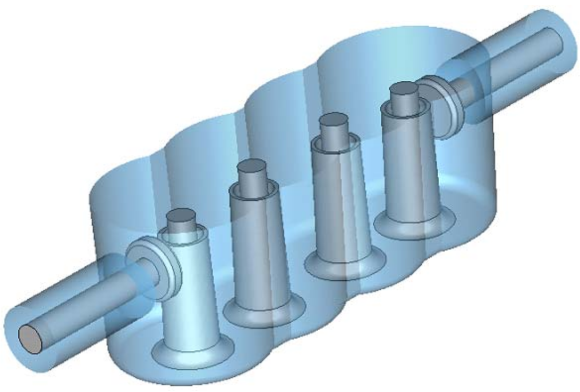
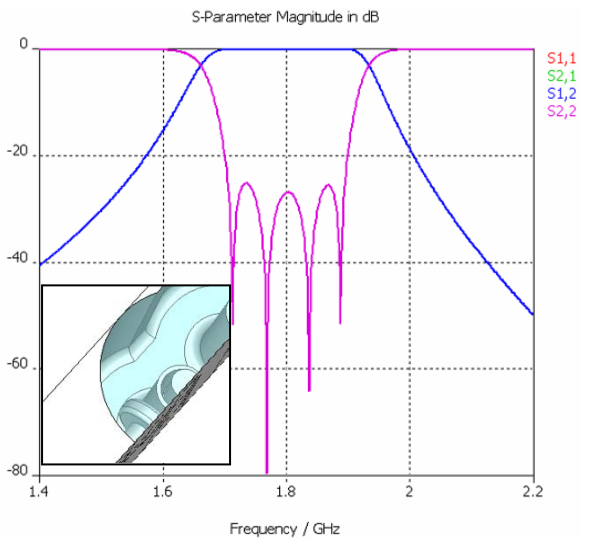


Fig. 9. 4-post pass-band filter. Model courtesy of the company Spinner GmbH, Westerham, Germany.



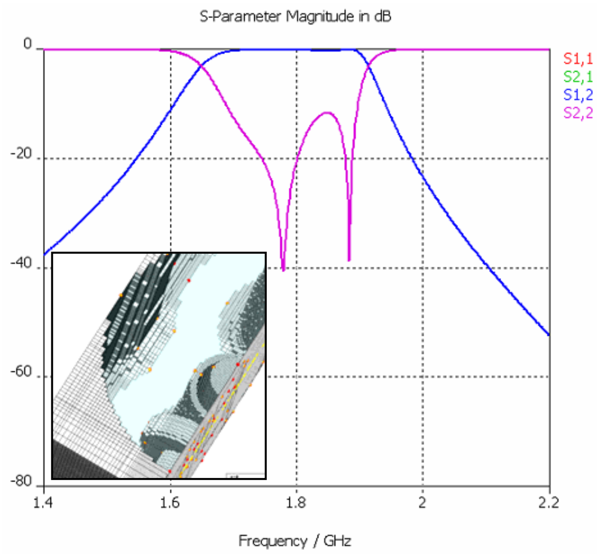


Fig.10. Simulated S-parameters for the 4-post pass-band filter. Upper plot: PBA mesh; lower plot: staircase mesh.

The PBA mesh (obtained based on an adaptive mesh refinement) contained about 50,000 mesh cells and the time needed for one simulation was of 5 minutes. The final S-parameters are shown in the upper plot of Fig. 10.

For the staircase mesh, the number of mesh cells needed was much higher, of about 1 million, and thus the computing time 5 times larger than in the PBA case – 25 minutes. However, the simulated results for the tuned

References

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[2] T. Weiland, "Time Domain Electromagnetic Field Computation with Finite Difference Methods", *International Journal of Numerical Modelling*, vol. 9, pp. 295-319, 1996.

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[5] J. B. Ness, "A Unified Approach to the Design, Measurement, and Tuning of Coupled-Resonator Filters", *IEEE Trans on Microwave Theory and Tech*, vol. 46, no. 4, April 1998.