3D Field Simulations using FI Time Domain Technique of Wedge- and Parabolic-Shaped Left Handed Materials (LHM)

Franz Hirtenfelder(1), Grzegorz Lubkowski(2)

(1) CST GmbH, Bad NauheimerStrasse 19, D-64289 Darmstadt, Germany
Email:franz.hirtenfelder@cst.com

(2) Techn. Univ. Darmstadt, TEMF, Schloßgartenstrasse 8, D-64289 Darmstadt, Germany
Email:lubkowski@temf.tu-darmstadt.de

ABSTRACT

Wedge- and parabolic shaped structures composed of split-ring resonators (SRR) and wires are numerically simulated by evaluating its refraction and radiation behaviour using CST MICROWAVE STUDIO®'s Finite Integration Time Domain Solver. Negative phase velocity inside these metamaterials is demonstrated in the left-handed band and the Snell's Law is conformed in terms of its refraction behaviour. Effective electric permittivity and magnetic permeability for these metamaterials are extracted directly from S-parameters of a SRR unit cell described with Lorentz/Drude dispersion models. It is demonstrated that both, models composed of a homogeneous Left-Handed-Material (LHM) and models consisting of a large number of microscopic unit-cells show similar results. Antennas designed with metamaterials offer the ability to adjust phase center easily thereby showing a larger constant phase angle.

INTRODUCTION

Metamaterials are a new class of man-made materials that can be engineered to respond to EM fields in unconventional ways, described by the permittivity ε and permeability μ. Depending on these two values some materials exhibit a negative refractive index, particularly when both ε and μ are negative (double negative DNG) at certain frequencies. One of the concepts for the construction of a DNG metamaterial is to use a combination of split ring resonator (SRR) providing a negative permeability μ and of wires providing a negative ε, respectively. Metamaterial structures are built of periodically ordered unit cells with the assumption that the lattice constant is less than the wavelength in this medium. The effective ε and μ can be deduced by averaging the electromagnetic field in a cell with appropriate boundary conditions. Another method is to calculate them by analyzing the reflection and transmission S-parameters. The shape of parameterized characteristics for effective ε and μ is assumed a priori and their parameters are optimized in order to obtain the best fitting to the reference response. Numerical evaluation of LHM is valuable since it can describe the refracted and propagated field distributions more clearly and can deliver the phase information inside the composite material. In this paper wedge- and parabolic shaped metamaterials are investigated and simulated numerically for both methods, the real composite metamaterial and the continuous media with derived material properties. It is shown, that within time domain simulations even large, complicated periodic structures can be analyzed.

PARAMETER FITTING OF DISPERSION MODELS

The method proposed here tries to minimize the difference between the scattering parameters obtained for the reference structure – the real detailed periodic structure containing e.g. SRRs and wires- and a homogeneous structure – a block of isotropic homogeneous dispersive material described by dispersive Drude (ε) and Lorentz (μ) models [1]. The analytic equation of effective ε for Drude is given by

$$\varepsilon_{eff}(\omega) = \varepsilon_\infty - \frac{\omega_p^2}{\omega(\omega - i\nu_c)} \tag{1}$$

where ε_∞ is the electric permittivity at the high frequency limit, ω_p the radial plasma frequency, ν_c the collision frequency. The analytic equation of effective μ for Lorentz is given by

$$\mu_{eff}(\omega) = \mu_\infty + \frac{(\mu_s - \mu_\infty)\omega_0^2}{\omega_0^2 + i\omega\delta - \omega^2} \tag{2}$$

where μ_s is the permeability at the low and μ_∞ at the high frequency limit, ω_0 the radial resonant frequency and δ the damping constant. A Quasi-Newton optimization algorithm built into CST MICROWAVE STUDIO®'s optimizer [2],[3] is used to search for the parameters of (1) and (2) providing the best fit of the S-parameters of the homogeneous model and the unit cell structure. The form of the optimizer goal function to be minimized takes the form of:

$$Goal = \sum_n (\left|S_{11} - S_{11ref}\right| + \left|S_{21} - S_{21ref}\right| + \left|\angle S_{11} - \angle S_{11ref}\right| + \left|\angle S_{21} - \angle S_{21ref}\right|) \quad (3)$$

where the fit runs over all n frequency samples in the range of interest.

The reference unit cell is given in Fig.1 and is similar to the one used in [4]. The structure contains two ports exhibiting the first dominant mode at both ports located left and right of the cubical vacuum cell. The top and bottom boundaries are assigned to perfect electric, the front and rear side to perfect magnetic boundary conditions. For the homogeneous model the wires and SRRs are removed and the extended slab covering the whole cubical block is assigned to a Lorentz/Drude material definition. As additional optimization parameters the deembedding phase could be used to adjust the reference phase. A comparison of S-parameters obtained by the optimization process with the reference results in Fig.2 shows a very good agreement. The optimized parameters of Lorentz/Drude are given in Tab. 1.

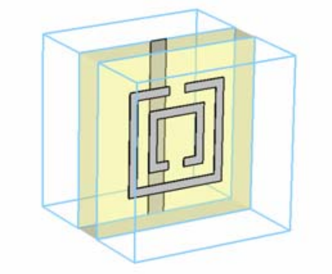

Fig.1 Unit cell reference structure

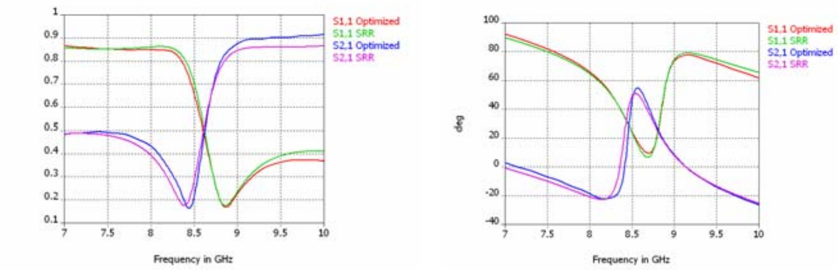

Fig .2 Magnitude and Phase of the S-parameter fit after the optimization

With the obtained parameter values of Lorentz/Drude models, the effective parameters for ε and μ given in (3) show a LHM behaviour in the frequency range 8.5 – 9 GHz, this is also confirmed by the negative propagation factor β exhibit in Fig.4.

Parameter	Optimized Values
ε_∞	2.01881
ω_p	2*π*14.16 GHz
υ_c	30.8075 MHz
μ_s	1.273475
μ_∞	1.12543
ω_∞	2*π*8.47342 GHz
δ	1.2579 GHz

Tab. 1 Optimized parameter values for the SRR/wires structure

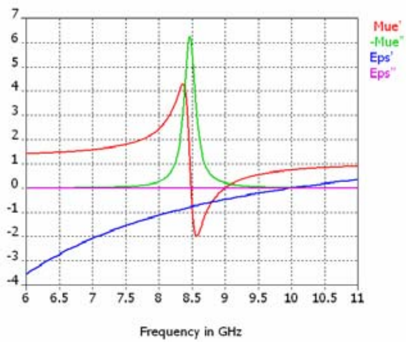

Fig. 3 Effective magnetic permeability and permittivity obtained by parameter fitting

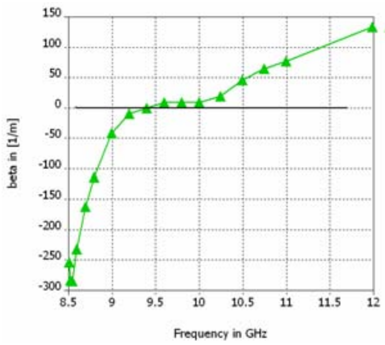

Fig. 4 Phase propagation for the bulk dispersive material. The range between 8.5 and 9 GHz exhibits a negative β

THE WEDGE MODEL

Using the continuous media is a very convenient way to apply this LHM material to more complex type of geometries like a wedge, where the negative index of refraction is demonstrated. Fig.5 is composed of a slice of a LHM wedge fed by a waveguide. Top and bottom boundaries are assigned to periodic boundaries with a phase shift of zero, the remaining sides are assigned to open boundaries. The model exhibits a LHM band around 8.5 – 9 GHz as plotted in Fig. 6. In a second step, the lossy bulk material is replaced by the SRR unit cells at the lateral plane. The size of these cells has to be smaller than the guided wavelength. The refraction interface has a staircase pattern with a step ratio of 2:1 which can be referred as a wedge of 26.6 deg. Even with this more complex geometry the time domain simulation performed with CST MICROWAVE STUDIO® was less than 11 min. Figs. 7 and 8 show the composite material embedded in a feeding waveguide. For demonstrating the negative refraction index, a transversal E-field monitor at 8.56 GHz shows the refracted field distribution with a refraction direction angle of 25 deg. Applying Snell's Law the refraction index n = sin 26.6deg / sin –(22) deg = -1.17. At a somewhat higher frequency (9.8 GHz) where ε is negative and μ is positive, also accompanied by a strong attenuation of the transmitted signal and/or large reflection, the refraction angle is very low and the direction is nearly parallel to the normal of the refraction interface: n = sin 26.6deg /sin5deg = 5.14. Both effects are illustrated for the transversal E-field at Figs. 9 and 10, respectively.

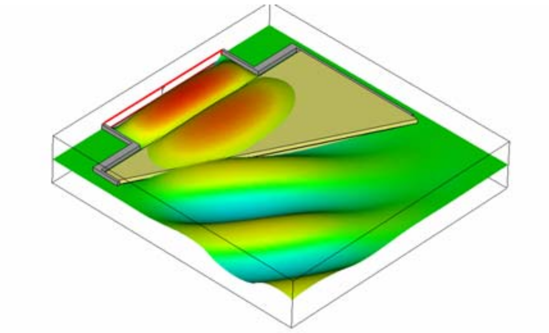

Fig. 5 E field carpet plot of the bulk LHM wedge model

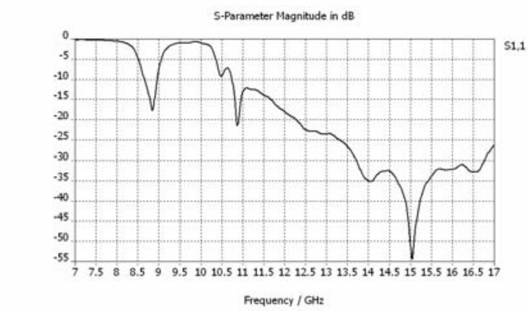

Fig.6 Reflection Parameter including a LHM band

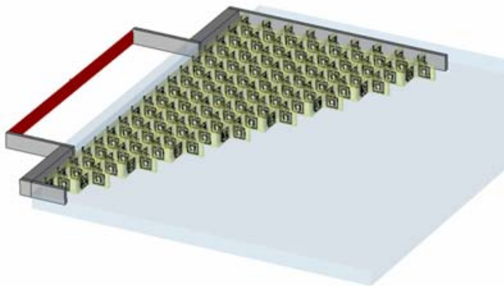

Fig 7 The simulated configuration with a wedge of 26.6 º

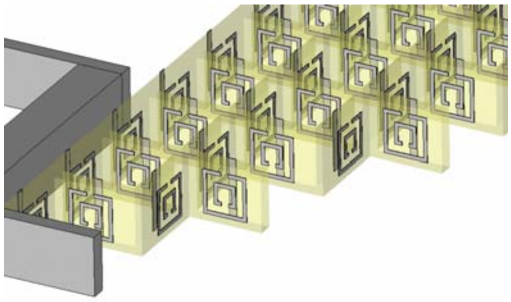

Fig 8 Close up look at the SRRs and wires

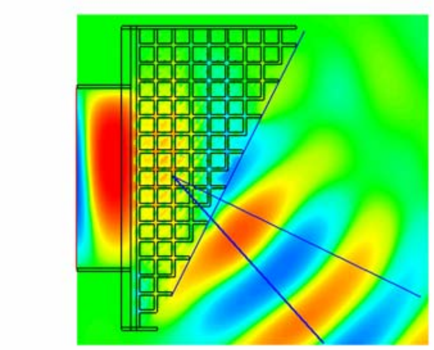

Fig. 9 The negative refraction in the LHM band

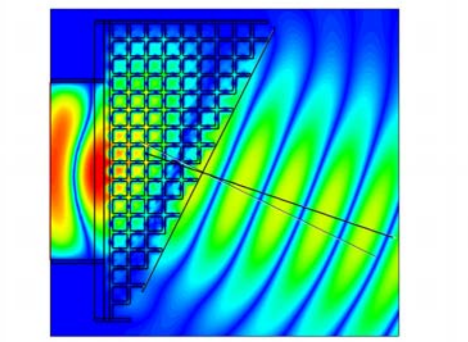

Fig. 10 Positive refraction in the low-index band

THE PARABOLIC ANTENNA MODEL

A further "deformation" of the planar wedge refraction interface into a parabolic shape may lead to a type of focusing antenna with unusual phenomenon. With a negative index of refraction, the material bends an incident wave away from the angle of incidence, instead of towards. This allows focussing of waves and the fabrication of antennas that behave electrically as if they are larger than in reality. In a first design a parabolic shape consisting of bulk LHM material with a focus length of 50 mm was constructed. Due to the negative refraction, the actual phase-center is located somewhat closer, in this particular case at around 36 mm as shown in Fig. 11 for the directivity. Note, that the LHM material is highly lossy and the bandwidth of negative refraction index is narrow. In a final simulation model the composite material has been used. The "PBA™ meshing technique of CST-MICROWAVE STUDIO ® [5] allows a relatively coarse meshdensity without loosing the macroscopic LHM effect. The model consists of about 1 Mio meshcells –detail view is shown in Fig. 13- and required 3 h of CPU time (PC 2.66GHz) with a 600MB RAM for the broadband (5-20 GHz) time domain simulation. Since the parabolic shape is mimicked with unit cell SRRs/wires, the center part of the parabol remains flat, thus the phase center of 12 mm is located closer towards the aperture, depicted in Fig. 12. Phase propagation plots are given in Fig. 14 to demonstrate a reversed phase velocity inside the LHM. Furthermore, the constant phase angle is much wider than for antennas without LHM materials, as demonstrated in Fig.16.

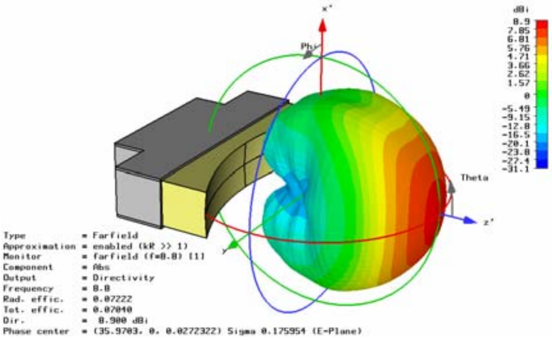

Fig 11 Farfield directivity for the Bulk LHM material model with a computed phase center at 36 mm

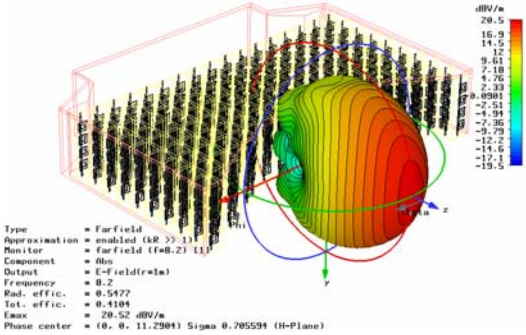

Fig 12 Farfield-E for the composite setup of the antenna with a computed phase center at 12 mm

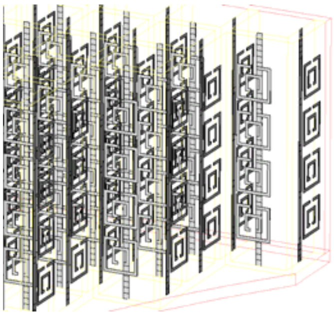

Fig 13 close-up look at the SRRs and wires

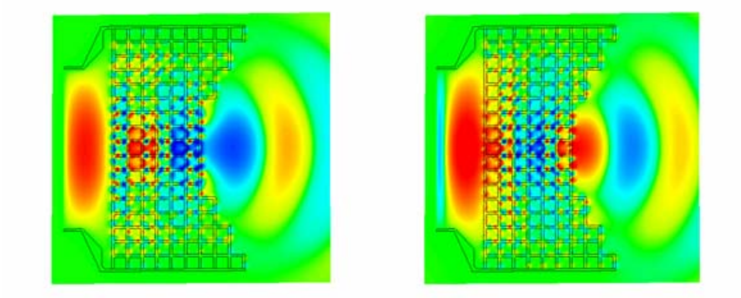

Fig 14 E-Field plots at two instances of phase.

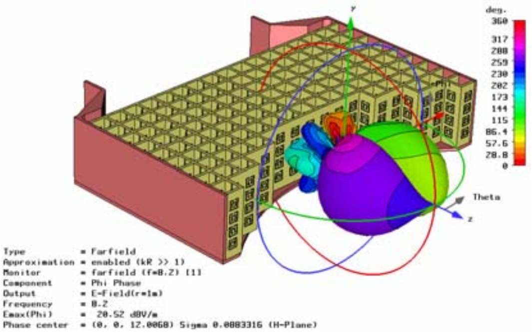

Fig 15 Phase plot for the vertical E-field

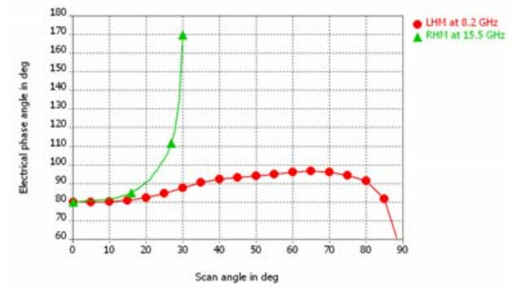

Fig 16 Comparison of constant phase angle width

CONCLUSION AND SUMMARY

In summary we report here our simulation results on a wedge- and parabolic- shaped composite and continuous metamaterial. Negative phase and refraction indices inside the metamaterials is exhibited inside the left-hand band in which permittivity and permeability are both negative. Numerical evaluation of such experiments is valuable since it can describe the refracted field distributions more clearly together with phase information inside the composite material.

REFERENCES

[1] G. Lubkowski, R. Schuhmann, T. Weiland, "Extraction of effective metamaterial parameters by parameter fitting of dispersive models", submitted for publication in *Microwave and Optical Technology Letters*.

[2] T. Weiland, " A discretization method for the solution of Maxwell's equations for six-component fields", *Electronics and Communication*, (AEÜ), Vol. 31 1977, . p.116

[3] T. Weiland, "Time Domain Electromagnetic Field Computation with Finite Difference Methods", *International Journal of Numerical Modelling*, Vol. 9, pp. 295-319, 1996

[4] R.A. Shelby, D.R. Smith and S. Schultz, "Experimental verification of a negative index of refraction", *Science 292(5514) (2001)*, 77-79.

[5] B. Krietenstein, R. Schuhmann, P. Thoma and T. Weiland, „The Perfect Boundary Approximation Technique facing the challenge of high precision field computation", *Proc. of the XIX International Linear Accelerator Conference (LINAC 98)*, Chicago, USA, 1998, pp. 860-862